AIME Counting course assessment test

Da Vinci Math

Don't expect to get all questions correctly. You can take this course if you can get the first 5 problems correctly without any help.

1. Place 6 balls into 3 bins, how many different ways if the balls are indistinguishable and bins are distinguishable. Bins can be empty.

2. Place 6 balls into 3 bins, how many different ways if the balls are indistinguishable and bins are indistinguishable. Bins can be empty.

3. Place 6 balls into 3 bins, how many different ways if the balls are distinguishable and bins are indistinguishable. Bins can be empty.

4. Place 6 balls into 3 bins, how many different ways if the balls are distinguishable and bins are distinguishable. Bins can be empty.

5. Place 6 balls into 3 bins, how many different ways if the balls are distinguishable and bins are distinguishable. Bins can't be empty.

6. Hang 6 balls onto 3 ropes on the ceiling, how many different ways if the balls are distinguishable and ropes are distinguishable. Ropes can't be empty.

7. Four leaves are in place and a cricket is sitting on one leaf. Each move the criket will hop from its current leaf onto any other leaf with same chance. What is the probability that the cricket comes back to the original leaf after 6 hops.

8. We place triangles and circles on a 4×4 board. Each block must be filled with a triangle or a circle. How many ways to fill these 16 blocks such that there is at least 1 row/column with all triangles and 1 row/column with all circles.

9. We place triangles and circles on a 4×4 board. Each block must be filled with a triangle or a circle. How many ways to fill these 16 blocks such that every row and every column the number of triangles (diagonal row does not count) is an even number (could be 0).

10. A cubic cake is on the table, if you are allowed to make 4 cuts on the cake, what is the maximum number of pieces you can get?

Q1 28

Q2 7

Q3 122

Q4 729

Q5 $3^6 - 3 \cdot 2^6 + 3 = 540$

Q6 7200

Q7 $\frac{61}{243}$. Use recursion $P_{n+1} = \frac{1}{3}(1 - P_n)$.

Q8 5404

Q9 512

Q10 This question be solved with recursion. Let f_n be the largest number of pieces with n cuts. The "new cut" is a planary cross-section on the cube. To maximize the number of pieces, the $n + 1$th cut must intersect all the previous cuts. The intersection of two cuts leaves a "crease" on the cross-section. Therefore, on the new cross-section there are at most n creases which divide the plane into $\frac{n(n+1)}{2} + 1$ number of regions. These regions divide the piece of cube where they cross into two pieces. Therefore, the recursion is

$$f_{n+1} = f_n + \frac{n(n+1)}{2} + 1.$$